EECE 290 – Quiz3 Solution

April 18, 2015

**A.** Determine *vC*(0+) and *i*(0+), assuming the capacitor is initially

charged to *Vc*0 = 2 V.

**Solution:***vC*(0+) = *VC*0 since the capacitor voltage is not forced to change. = 5 – *Vc*0/2 A.

**Version 1:***Vc*0 = 2 V; *vC*(0+) = 2 V, *i*(0+) = 5 – 2/2 = 4 A

**Version 2:***Vc*0 = 3 V; *vC*(0+) = 3 V, *i*(0+) = 5 – 3/2 = 3.5 A

**Version 3:***Vc*0 = 4 V; *vC*(0+) = 4 V, *i*(0+) = 5 – 4/2 = 3 A

**Version 4:***Vc*0 = 5 V; *vC*(0+) = 5 V, *i*(0+) = 5 – 5/2 = 2.5 A

**Version 5:***Vc*0 = 6 V; *vC*(0+) = 6 V, *i*(0+) = 5 – 6/2 = 2 A.

**B.** Determine *i*(0+) and *vL*(0+), assuming an initial current *I*0 =

2 A through the inductor.

**Solution:**The impulse appears across the inductor, establishing a flux linkage of 10 Vs across the inductor, which adds a current of 10/2 = 5 A to the initial inductor current. Hence, *i*(0+) = 5 + *IL*0 = 7 A. At *t* = 0+, *i* flowing through the resistor establishes a voltage 2(5 + *IL*0) = 10 + 2*IL*0 V = -*vL*(0+).

**Version 1:***IL*0 = 2 A; *i*(0+) = 5 + *IL*0 = 7 A, *vL*(0+) = -(10 + 2*IL*0 ) = -14 V

**Version 2:***IL*0 = 3 A; *i*(0+) = 5 + *IL*0 = 8 A, *vL*(0+) = -(10 + 2*IL*0 ) = -16 V

**Version 3:***IL*0 = 4 A; *i*(0+) = 5 + *IL*0 = 9 A, *vL*(0+) = -(10 + 2*IL*0 ) = -18 V

**Version 4:***IL*0 = 5 A; *i*(0+) = 5 + *IL*0 = 10 A, *vL*(0+) = -(10 + 2*IL*0 ) = -20 V

**Version 5:***IL*0 = 6 A; *i*(0+) = 5 + *IL*0 = 11 A, *vL*(0+) = -(10 + 2*IL*0 ) = -22 V.

**C.** Sketch the convolution of *f*(*t*) with *δ*(*t* – 0.5). (Just show the result, no need

for explanations).

**Solution:***f*(*t*) is delayed by the delay of the impulse.

**Version 1:**Delay = 0.5

**Version 2:**Delay = 1

**Version 3:**Delay = 1.5

**Version 4:**Delay = 2

**Version 5:**Delay = 2.5.

**D.** Derive the Laplace transform of 2*u*(*t*) + 3*δ*(*t* – 1).

**Solution:**

**Version 1:**

**Version 2:**

**Version 3:**

**Version 4:**

**Version 5:**.

**1.** Determine *vC*(0+), assuming *C* = 0.1 F and the capacitor

initially uncharged.

1. 6 V
2. 10 V
3. 20 V
4. 12 V
5. 8 V

**Solution:**The ideal voltage source and the resistor have no effect on the current impulse, which deposits 2 C on the capacitor, resulting in a voltage *vC*(0+) = 2/*C* V.

**Version 1:***C* = 0.1 F, *vC*(0+) = 2/*C* = 20 V

**Version 2:***C* = 0.2 F, *vC*(0+) = 2/*C* = 10 V

**Version 3:***C* = 0.4 F, *vC*(0+) = 2/*C* = 5 V

**Version 4:***C* = 0.5 F, *vC*(0+) = 2/*C* = 4 V

**Version 5:***C* = 0.8 F, *vC*(0+) = 2/*C* = 2.5 V.

**2.** The switch is closed at *t* = 0 after being

open for a long time. Determine *vO*(*t*),

*t*≥ 0+, assuming *VSRC* = 8 V.

1. 4 V
2. , *t* in ms
3. 6 V
4. , *t* in ms
5. 8 V

**Solution:** At *t* = 0-, one capacitor is charged to *VSRC*, the other is uncharged. At *t* = 0+, the charge is equally divided between the two capacitors, so that *vO*(0+) = *VSRC*/2. As *t*→∞, *vO*(∞) = *VSRC*/2, from voltage division. It follows that *vO*(*t*) = *VSRC*/2.

**Version 1:***VSRC* = 8 V; *vO*(*t*) = 4 V

**Version 2:***VSRC* = 10 V; *vO*(*t*) = 5 V

**Version 3:***VSRC* = 12 V; *vO*(*t*) = 6 V

**Version 4:***VSRC* = 14 V; *vO*(*t*) = 7 V

**Version 5:***VSRC* = 16 V; *vO*(*t*) = 8 V.

**3.** The switch is closed at *t* = 0, with the capacitors having

the initial voltages indicated. Determine the charge

delivered by the source, assuming *VSRC* = 10 V.

1. 12 μC
2. 14 μC
3. 16 μC
4. 10 μC
5. 18 μC

**Solution:**The capacitance of the two 5 μF capacitors in series

is 2.5 μF. This in series with 10 μF gives *Ceqs* = (2.5×10)/12.5 =

2 μF. The voltage across *Ceqs* = 5 V so that its charge is 10 μC.

After the switch is closed the charge on *Ceqs* is 2*VSRC* C. The charge

delivered by the source is therefore *q* = (2*VSRC* – 10) μC.

**Version 1:***VSRC* = 10 V; *q* = (2*VSRC* – 10) = 10 μC

**Version 2:***VSRC* = 11 V; *q* = (2*VSRC* – 10) = 12 μC

**Version 3:***VSRC* = 12 V; *q* = (2*VSRC* – 10) = 14 μC

**Version 4:***VSRC* = 13 V; *q* = (2*VSRC* – 10) = 16 μC

**Version 5:***VSRC* = 14 V; *q* = (2*VSRC* – 10) = 18 μC.

**4.** The switch is closed at *t* = 0, with *C*1initially chargedto *V*10=2 V.

Determine the initial voltage *V*20of *C*2that will make **each** of *C*1 and *C*2 completely discharge as *t*→∞.

1. 6 V
2. 9 V
3. 12 V
4. 15 V
5. 3 V

**Solution:** The initial charge on *C*1 is 3×2 = 6μC. If *C*2 is to discharge completely when *C*1 discharges, *C*2 must also have an initial charge of 6μC. It follows that *V*20 = 6/2 = 3 V.

*Check*: *Ceqs* = 3×2/5 = 1.2 μF. The voltage across *Ceqs*is 5 V and *Qeqs* = 1.2×5 = 6μC. When *Ceqs* discharges, 6μC flow through *C*1and *C*2, completely discharging each of them.

**Version 1:***V*10 = 2 V; *V*20 = (2×3)/2 = 3 V

**Version 2:***V*10 = 4 V; *V*20 = (4×3)/2 = 6 V

**Version 3:***V*10 = 6 V; *V*20 = (6×3)/2 = 9 V

**Version 4:***V*10 = 8 V; *V*20 = (8×3)/2 = 12 V

**Version 5:***V*10 = 10 V; *V*20 = (10×3)/2 = 15 V.

**5.** Determine the maximum value of the

convolution integral *f*(*t*)\**g*(*t*) and the

time at which it occurs, assuming *A* = 1.

1. 6 at *t* = 2
2. 3 at *t* = 2
3. 3 at *t* = 3
4. 6 at *t* = 3
5. 3 at *t* = 6

**Solution:**When *f*(*t*) is folded around the vertical axis and shifted by 2 units to the right, *f*(*t*) and *g*(*t*) are aligned. The area under the product is a maximum and equals *A*×3×2/2 = 3*A*.

**Version 1:***A* = 1; *y*max = 3*A* = 3 at *t* = 2

**Version 2:***A* = 2; *y*max = 3*A* = 6 at *t* = 2

**Version 3:***A* = 3; *y*max = 3*A* = 9 at *t* = 2

**Version 4:***A* = 4; *y*max = 3*A* = 12 at *t* = 2

**Version 5:***A* = 5; *y*max = 3*A* = 15 at *t* = 2.

**6.** Derive the Laplace transform of *tu*(*t* – 2).

1. 
2. 
3. 
4. 
5. 

**Solution:***Atu*(*t* – 2) = *A*(*t*– 2)*u*(*t* – 2) + 2*Au*(*t* – 2). The LT is .

**Version 1:***A* = 1; LT = 

**Version 2:***A* = 2; LT = 

**Version 3:***A* = 3; LT = 

**Version 4:***A* = 4; LT = 

**Version 5:***A* = 4; LT = .

**7.** In the circuit shown, the initial voltage

on the capacitor is *V*10 = 6 V, and the

initial current in the inductor is

*I*20 = 2 A. Determine *i*1(0+), *i*2(0+),

*v*1(0+), and *v*2(0+) (5 grades each).

**Solution:** Initially, when the impulse is applied,

the capacitor behaves as a short circuit and the

inductor as an open circuit. The impulse

divides between the two parallelpaths, with an impulse of 4*δ*(*t*) A flowing through

the two 1 Ω resistors. The impulse deposits

a charge of 4 C on the capacitor, which adds at *t* = 0+ to the initial voltage across the capacitor, so that *v*1(0+) = 10 V. The 4*δ*(*t*) A flowing through the 1 Ω resistor in parallel with the inductor produces a voltage impulse 4*δ*(*t*) V

across the 2 H inductor, which establishes a flux linkage

of 4 Vs in the inductor, resulting in a 2 A current through the inductor. Added to the initial 2 A current, this makes *i*2(0+) = 4 A. The circuit at *t* = 0+ becomes as shown. The current in the 1 Ω resistor inparallel with the current source is *v*2 A, and the current through the 1 Ω resistor in series with the voltage source is (*v*2 + 4) A. From KVL around the mesh, 5(*v*2 + 4) + 10 + *v*2 = 0, which gives *v*2(0+) = -5 V, and *i*1(0+) = *v*2 + 4 = -1 A.

**8.** The switch is closed at *t* = 0, with *C*1 initially charged

to*V*10 = 9 V, and *C*2 and *C*3 uncharged. Determine:

(a) *v*1(0+), considering that the voltage across *C*3 is

not forced to change at *t =* 0*+*(7 grades)*;* (b)*i*(*t*) for *t*≥ 0+(7 grades); (c) the voltage across *C*3 as *t*→∞, by evaluating the charge deposited on *C*3 by *i* as *t*→∞ (6 grades).

**Solution:** (a) At *t* = 0+, the following holds: i) the voltage across *C*1 and *C*2 is equalized to satisfy KVL; ii) the voltage across *C*3 stays at zero, since KVL across the *C3R* branch can be satisfied by the *Ri* voltage drop; iii) although *i* flows at *t =* 0*+*,no charge yet flows, so that charge at the upper plates of *C*1 and *C*2 is conserved. This charge is initially 4×9 = 36μC. At *t =* 0*+* this charge is distributed on *C*1 and *C*2 in parallel, resulting in a voltage *v*1(0+) = 36/(4 + 2) = 6 V.

(b) *i*(0+) = 6/1 = 6 mA, and *i*→ 0 as *t*→∞; the capacitance seen by the 1 kΩ resistor is (3×6)/(3 + 6) = 2 μF, so that *τ* = 1×2 = 2 ms. It follows that mA where *t* is in ms.

(c) The charge deposited by *i* on *C*3 is: μC. The final voltage on *C*3 is 12/3 = 4 V. This is the same voltage derived from the initial charge of 36 μC distributed between *C*1, *C*2, and *C*3 in parallel, having a capacitance of 9 μF, so that the final voltage is 36/9 = 4 V. The initial charge of 36 μC is first distributed at *t* = 0+ between *C*1 and *C*2, and eventually between *C*1, *C*2, and *C*3 in parallel as *t*→∞. Charge is conserved at the upper plates of the three capacitors.

**9.** Convolve *f*(*t*) and *g*(*t*), where *f*(*t*) is a pulse of 2 units

amplitude and 1 unit duration, and *g*(*t*) consists of two sinusoidal half cycles represented by: *g*(*t*) = sin*πt*, 0 ≤*t*≤ 1, and *g*(*t*) = -sin*πt*, 1 ≤*t*≤ 2.

**Solution:***t* is replaced by *λ* in both functions, *f*(*λ*) is folded and shifted to

the right of *t*. as shown. The ranges of *t* are:

0 ≤*t*≤ 1, 

1 ≤*t*≤ 2, 



. Note that

this twice the area under half a sinusoid.

Check: at *t* = 1, *y*1(*t*) = 4/*π* = *y*2(*t*).

1 ≤ (*t* – 1) ≤ 2, or 2 ≤*t*≤ 3, 



. Check: At *t* = 2, *y*3(*t*) = 4/*π* = *y*2(*t*), and at *t* = 3, *y*3(*t*) = 0, since there is no overlap. The convolution integral will be as shown.

**Analytical convolution:***f*(*t*) = 2*u*(*t*) – 2*u*(*t* – 1); the first half-sinusoid can be represented as sin*πtu*(*t*)+ sin*π*(*t* – 1)*u*(*t* – 1); since the second half-sinusoid is the first half-sinusoid delayed by 1 unit; it can be represented as:sin*π*(*t* – 1)*u*(*t* – 1) + sin*π*(*t* – 2)*u*(*t* – 2).Hence: *y*(*t*) = [2*u*(*t*) – 2*u*(*t* – 1)]\*[sin*πtu*(*t*) +2sin*π*(*t* – 1)*u*(*t* – 1) + sin*π*(*t* – 2)*u*(*t* – 2)]. It follows from Equation 20.4.9 that: –. The integrals multiplying the various step functions are evaluated as follows: ; ; ; . The convolution integral is therefore given by:

. It is seen that for 0 ≤*t*≤ 1, *y*(*t*) is equal to the term multiplied by *u*(*t*), which is ; for 1 ≤*t*≤ 2, *y*(*t*) is equal to the sum of the terms multiplied by *u*(*t*) and *u*(*t* – 1), which gives ; for 2 ≤*t*≤ 3, *y*(*t*) is equal to the sum of the terms multiplied by *u*(*t*), *u*(*t* – 1), and *u*(*t* – 2), which gives ; for*t*≥ 3, *y*(*t*) is equal to the sum of all the terms, which is zero.